

# Thinnest optical waveguide: experimental test

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A thin dielectric waveguide with a subwavelength diameter can exhibit very small transmission loss only if its diameter is greater than a threshold value, while for smaller diameters, waveguide loss grows dramatically. The threshold diameter of transition between these waveguiding and nonwaveguiding regimes is primarily determined by the wavelength of propagating light and, to a much lesser degree, by the characteristic length of the waveguide's long-range nonuniformity. For this reason, the transmission spectrum of a thin waveguide allows immediate and quite accurate determination of its thickness. An experimental test of these facts is performed for a tapered microfiber. Good agreement with the recently developed theory of adiabatic microfiber tapers is demonstrated. © 2007 Optical Society of America  
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Theoretically, the fundamental mode of a uniform lossless dielectric waveguide exists independently of its thickness. However, in practice, waveguiding ability is constrained by losses due to material absorption and geometric nonuniformities. Here we apply the term “waveguide” only to such a waveguide that exhibits losses that are small in comparison with the input power. Subsequently, a basic, practical question can be asked: how thin can the optical waveguide be?<sup>1</sup> It is crucial to note that consideration of waveguide losses is incomplete if input and output losses are ignored. From this point of view, the transmission loss of a waveguide is determined by its own optical properties, as well as by the way the input and output of light is performed. The choice of input and output solutions (which may or may not be supported by waveguides) is necessary for the determination of transmission properties of a waveguide. There exists, in most cases, a way to minimize input and output losses down to relatively small values. For this reason, these losses often (as, e.g., in optical fiber communications) can be separated from the losses of the waveguide itself. However, a very thin waveguide and, in particular, an optical microfiber (MF) is an exception to this rule: Its transmission loss is primarily determined by input and output losses, which, in practice, cannot be reduced significantly.<sup>1,2</sup> Previous experimental results<sup>3–5</sup> indicate the existence of a steep transition between the highly lossless and highly lossy regimes in MF transmission. However, to the best of our knowledge, this effect has not been fully understood and analyzed experimentally. This Letter shows that the experimentally determined diameter of the thinnest MF waveguide is in good agreement with predictions of the recently developed theory of adiabatic MF tapers.<sup>6</sup> Also, a simple method of determining the MF diameter from its transmission spectrum is demonstrated.

The evanescent field outside a very thin MF with diameter  $d$ , which is significantly less than the radia-

tion wavelength,  $\lambda$ , behaves as  $K_0(\gamma\rho)\exp[(i(k + \gamma^2/2k)z)]$ . Here  $(\rho, z)$  are the cylindrical coordinates,  $k = 2\pi n_1/\lambda$  is the propagation constant in free space,  $n_1$  is the refractive index of the ambient medium,  $\gamma$  is the absolute value of the transversal component of the propagation constant, and  $K_0(x)$  is a modified Bessel function. For  $d \ll \lambda$ , the transversal propagation constant  $\gamma$  is defined by an asymptotic formula<sup>2,7</sup> which, for a silica MF with refractive index 1.45 in free space, gives

$$\gamma = \frac{3.31}{d} \exp\left(-0.285 \frac{\lambda^2}{d^2}\right). \quad (1)$$

In a tapered MF,  $\gamma$  is a function of coordinate  $z$ ,  $\gamma = \gamma(z)$ . Below we use a model of a taper with

$$\gamma(z) = \gamma_1 + \frac{\gamma_2 - \gamma_1}{1 + \exp(-z/L)}. \quad (2)$$

Equation (2) describes a conical taper, which monotonically changes along the characteristic length  $L$  having  $\gamma(z) = \gamma_1$  at  $z \rightarrow -\infty$  and  $\gamma(z) = \gamma_2$  at  $z \rightarrow +\infty$ . The asymptotic expression for the propagation loss of this taper can be found from Ref. 6 in the form:

$$P = \frac{k^{1/2} |\gamma_1 - \gamma_2|}{4L^{1/2} |\gamma_1 \gamma_2|} \exp\left[-\frac{\pi L}{k} \min(\gamma_1^2, \gamma_2^2)\right]. \quad (3)$$

If the taper has the diameters  $d_1$  and  $d_2$  at  $z \rightarrow \mp \infty$  then  $\gamma_1$  and  $\gamma_2$  in this equation are found from  $d_1$  and  $d_2$ , respectively, with Eq. (1). Equation (3) is valid only if the exponent is large and therefore  $P \ll 1$ . The condition  $P \sim 1$  defines the MF parameters, for which Eq. (3) fails, as well as the threshold diameter of a MF. From Eq. (3), if the tapering is relatively strong, then  $\gamma_1 \gg \gamma_2$ , and the loss is determined only by the characteristic length and the transversal propagation constant of the thinnest part of the taper:

$$P = \frac{\pi^{1/2}}{4S^{1/2}} \exp(-S), \quad S = \pi L \gamma_2^2 / k. \quad (4)$$

In our experiment, the MF represents a waist of a biconical tapered fiber fabricated using the indirect laser heating method.<sup>8</sup> The drawing method consists of periodical translations of the fiber into opposite directions with respect to the laser beam and simultaneous pulling in opposite directions.<sup>9</sup> In our drawing program, after a cycle of drawing, the fiber diameter is set to be reduced by a factor  $f=0.75$ , so that after the  $N$ th cycle the MF diameter is<sup>9</sup>

$$d^{(N)} = Df^N = 125 \cdot 0.75^N \mu\text{m}. \quad (5)$$

Curve 1 in Fig. 1 shows the diameter variation of the MF waist after 17 cycles of drawing, which was measured by the fiber-probe scanning method.<sup>10</sup> Curve 2 in this figure shows the MF profile after the drawing program has completed 17 cycles and was continued for a quarter of the next cycle, so that the laser beam was stopped in the center of the MF. The size of the center transition region in curve 2 defines the length of the MF, which is softened by the laser beam in the process of drawing. The transition region can be quite accurately fitted by the parametric function of Eq. (2) with  $L=250 \mu\text{m}$  (curve 3). Interestingly, the transition region between the uniform and nonuniform parts of the MF in curve 1 can be fitted with the parametric function of Eq. (2) having the same characteristic length parameter  $L=250 \mu\text{m}$  (curve 4). For larger  $N$ , the corresponding MF profiles can be simply rescaled from the profiles in Fig. 1, with the scale factor determined from Eq. (5). Using Eq. (1), we have found numerically that the transversal propagation constant dependence,  $\gamma(z)$ , has the same length parameter,  $L=250 \mu\text{m}$ , as the MF diameter variation.

At each half of a cycle of drawing the MF diameter is reduced by a factor of  $f^{1/2}=0.866$ , while, from Eq. (1), the transversal propagation constant for the MF diameters and wavelength of our interest is reduced

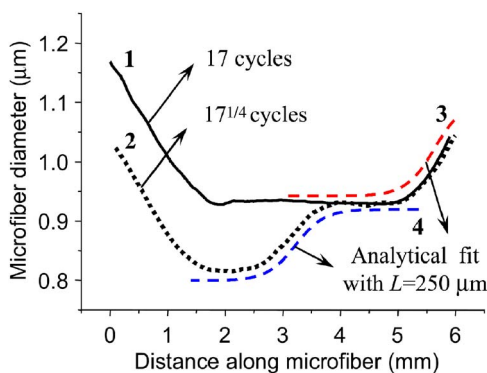


Fig. 1. (Color online) MF diameter variation found with the fiber-probe scanning method of Ref. 10. Curve 1, profile of a MF fabricated with 17 cycles of drawing; Curve 2, profile of a MF fabricated with  $17\frac{1}{4}$  cycle of drawing; Curve 3, analytical fit of curve 1 by the function given by Eq. (2) for  $L=250 \mu\text{m}$ ; Curve 4, analytical fit of curve 3 by the function given by Eq. (2) for  $L=250 \mu\text{m}$ . Curves 3 and 4 are shifted for visibility.

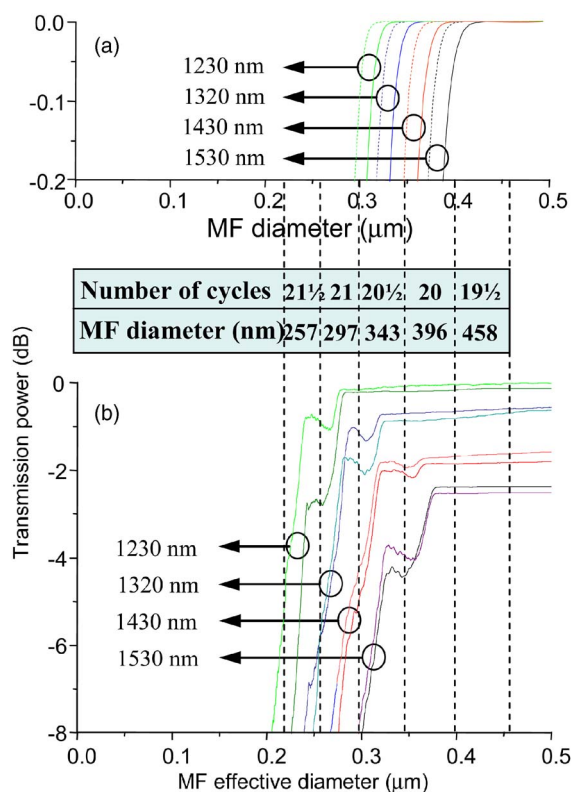


Fig. 2. (Color online) (a) Transmission loss as a function of MF diameter determined from Eq. (4) for different wavelengths (1230, 1320, 1430, and 1530 nm). (b) Transmission loss of a MF taper measured in the process of drawing for the same transmission wavelengths (1230, 1320, 1430, and 1530 nm) as a function of effective MF diameter. To illustrate the reproducibility of our measurements, two measurements for each wavelength are shown. The curves are shifted along the vertical axis for visibility. The table between figures shows the number of cycles and MF diameters corresponding to the regions separated by the vertical dashed lines.

by a factor of  $\sim 5$ . For this reason, in our modeling of the transmission loss of a MF, we ignore the larger  $\gamma_1$  in Eq. (3) and use Eq. (4). Then, adding the contributions of each of the two tapered regions, the transmission loss of our MF is found as twice the loss determined by Eq. (4). In Fig. 2(a), we plot this transmission loss as a function of the MF diameter for different wavelengths and determine the diameters, in the neighborhood of which the transmission loss starts the noticeable deviation from 0. The solid and dashed curves correspond to the MF nonuniformity  $L=250 \mu\text{m}$  and  $L=500 \mu\text{m}$ , respectively. The table below Fig. 2(a) determines the corresponding cycle numbers and the MF diameters found from Eq. (5). We tested the predictions of Fig. 2(a) experimentally by monitoring the transmission power of the MF taper in the process of drawing. Figure 2(b) shows the results of our measurements (to illustrate the reproducibility of measurements, two measurements corresponding to the same wavelength are shown). The horizontal axis in this figure is the effective MF diameter, which was calculated by the continuation of Eq. (5) to noninteger  $N$ . The transmission power in Fig. 2(b) corresponds to the actual MF diameters only

at the crosses of the vertical dashed lines and the horizontal axis. The characteristic step-shaped behavior of the curves near the threshold diameters is explained as follows. At the beginning of a cycle  $N$ , the MF diameter is tapered down to the diameter  $d^{(N)}$  in a small initial part of the MF, while the diameter of the rest of the MF remains equal to  $d^{(N-1)}$ . At this time, the strong reduction of transmission power (see, e.g., the curves corresponding to  $\lambda=1530$  nm and  $N=20$ ) may be observed. Next, the formed taper continues to move together with the laser beam along the MF as illustrated by curve 2 of Fig. 1, and the transmission power does not change significantly. Comparison of the theoretical prediction for the threshold diameters of Fig. 2(a) with the experimental data of Fig. 2(b) demonstrates a reasonable agreement of the threshold diameters. Notice that in the regions of the threshold transmission behavior, the losses caused by the reasons other than tapering (e.g., by material absorption and microbending) have much weaker dependence on the MF diameter and can be neglected. Figure 2(a) also shows that, while the threshold diameters strongly depend on the wavelength of light, they depend much more weakly on the characteristic nonuniformity of the MF,  $L$ : changing  $L$  by a factor of 2 modifies the threshold diameters only by less than 5%. Thus, for a fixed wavelength of light, the threshold diameter of a MF can be determined quite accurately with an order of magnitude estimate of the length of the MF long-range nonuniformity. For this reason, we believe that the threshold MF diameter should be also in a weak dependence on the actual profile of the MF taper, provided it is adiabatically smooth and qualitatively similar to the profile of biconical tapers considered here.

Similarly, as follows from Eqs. (1) and (3), for a MF of a fixed diameter, the threshold wavelength of propagating light can be determined, so that the MF has small losses below this wavelength, while, for larger wavelengths, the radiation losses grow rapidly. To show this, we fabricate a fiber taper by setting the drawing program to  $N=21$  cycles. From Eq. (5), the MF waist of this taper has a diameter  $d^{(21)}=297$  nm. The transmission spectrum of this MF is shown in Fig. 3. The inset in this figure enlarges the region of the spectrum where the propagation loss starts a noticeable departure from 0. In the inset, the experimental curve is compared with the theoretical dependences found from Eq. (4) with  $L=100, 250, 500,$  and  $1000 \mu\text{m}$ . The values of the MF diameter for these curves were chosen to arrive at the best fit with the experimental dependence for very small transmission losses. Though the chosen characteristic lengths,  $L$ , differ by up to a factor of 10, the difference between the corresponding threshold diameters is less than 9%, and their values are in reasonable agreement with the value  $d^{(21)}=297$  nm predicted by Eq. (5). Thus, the transmission spectrum of a MF allows a fairly simple and accurate estimate of its diameter.

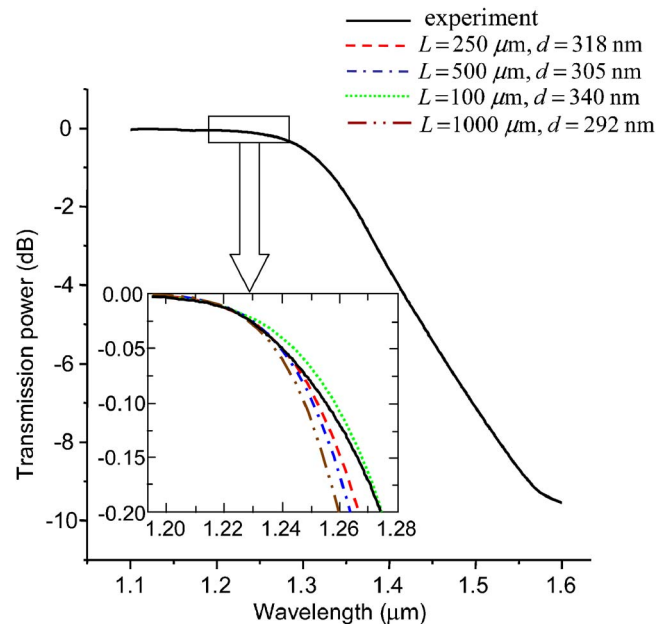


Fig. 3. (Color online) Transmission spectrum of a MF fabricated with 21 cycles of drawing. Inset: enlarged experimental spectrum and theoretical spectra found from Eq. (4). The parameters of theoretical spectra are shown in the figure.

In summary, a steep transition between highly lossless (waveguiding) and highly lossy (non-waveguiding) regimes in transmission of an optical MF is experimentally observed and theoretically explained. The crossover between these regimes is primarily defined by the wavelength of propagating light and the MF diameter. The experimentally determined threshold diameters (for the fixed wavelength of the propagating light) and threshold wavelength (for the fixed MF diameter) are shown to be in good agreement with the recently developed theory of adiabatic MF tapers.

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